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Computing the Average Renewal Reward in A Service System

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ABSTRACT

The liberty associated with any continuous Archimedean t-norm is investigated in this study for a renewal reward process with fuzzy random interarrival times and rewards. The renewal reward processes interarrival times and rewards are supposed to be positive fuzzy random variables with fuzzy realisations that are -independent fuzzy variables. In the following section, the long-run predicted reward per unit time of the renewal reward process is demonstrated using a fuzzy random renewal reward theorem. Finally, to demonstrate the utility of the result, some application examples are offered.

Keyword: Possibility Measure, Possibility Space, Credibility Measure, Renewal Processes, Renewal Reward Processes.

INTRODUCTION

Renewal incentive procedures are a type of renewal model that has a variety ofreal-world applications. The stochastic renewal reward processes create stochasticrenewal theory, which is based on probability theory. The interarrival times andrewards are assumed to be i.i.d random variables, and the stochastic renewal theoremis one of the most important results in this domain.In practical applications, it is acceptable to assume that statistic data inheritsuncertainty in the sense of vagueness due to subjective judgement, inaccurate humanknowledge, and perception. That is, when data is represented by fuzzy sets or fuzzyvariables rather than real numbers, ambiguity should be incorporated into statisticalanalysis techniques. Kwakernaak coined the term "fuzzy random variable" to describeinstances in which vagueness and randomness appear at the same time. Inspired bythe concepts of fuzzy random variable extensions and the strong law of largenumbers.A variety of real-world examples indicate that using the classical extensionconcept to combine fuzzy numbers is always the best option. Measurement theory,fuzzy control, and artificial neural networks are just a few examples. In some contextsand applications, the operations associated with various types of t-norms may berequired for fuzzy numbers. A general t-norm operator is used in a more generalextension principle. According to different t-norms, such a generalised extensionprinciple offers distinct procedures for fuzzy integers or fuzzy variables. Renewalreward processes with fuzzy random interarrival times and rewards that are T-independent for every continuous Archimedean t-norm. The renewal rewardprocessesinterarrival timings and rewards are supposed to be positive fuzzy randomvariables with fuzzy realisations that are T- independent fuzzy variables. For fuzzyrandom renewal rewards, various limit theorems in mean chance measure are derivedunder these conditions. In the following section, the long-run expected averagereward per unit time of renewal reward processes is proved using a fuzzy randomrenewal reward theorem.Using the class of continuous Archimedean t-norms, such asthe product t-norm and Yager t-norm, the renewal reward theorem isapplied to more generic scenarios in this study. Furthermore, instochastic processes, the conclusions derived in this paper degeneratenicely to the standard renewal reward theory.

BASIC DEFINITIONS

DEFINITION : 2.1(POSSIBILITY MEASURE)

Given a universe Γ, an **ample field**$\mathcal{A}$on $\Gamma$is a class of subsets of Γthat is closed under arbitrary unions, intersections, and complementation in Γ. Let Pos is a set function defined on the ample field $\mathcal{A}$. The set function Pos is said to be a **possibility measure**if it satisfies the following conditions:

(P1) $\mathrm{Pos}(\emptyset) = 0$, (P2) $\mathrm{Pos}(\Gamma) = 1$;

(P3) $\mathrm{Pos}\,(\cup_{i\in I} A_i) = \sup_{i\in I} \mathrm{Pos}\,(A_i)$ for any sub class $\{A_i|i\in I\}$ of $\mathcal{A}$, where I is an arbitrary index set.

DEFINITION :2.2 (POSSIBILITY SPACE)

The triplet $(\Gamma,\mathcal{A}, \mathrm{Pos})$ will be called a **possibility space**.

DEFINITION :2.3 (NECESSITY MEASURE)

The dual set function of possibility measure is called a **necessity measure**. It is denoted by Nec and it is defined by, $Nec(A) = 1 - Pos(A^c)$, where A^c is the complement of A

DEFINITION : 2.4 (CREDIBILITY MEASURE)

A self-dual set function is called **credibility measure**, It is denoted by Cr, and it is defined by,

$Cr(A) = (Pos(A) + Nec(A)) /2$ for any $A \in \Gamma(1)$

DEFINITION :2.5 (FUZZY VARIABLE)

Let $\Re$ be a set of all real numbers . A function $Y:\Gamma\rightarrow\Re$ is said to be a **fuzzyvariable**defined onΓ, and the possibility distribution μ_Y of Y is defined by

$\mu_Y(t) = Pos\{Y=t\}, t\in\Re$, which is the possibility event $\{Y=t\}$.

A fuzzy variable Y is said to be positive **almost surely** ,if $Cr\{Y \leq 0\}$.

DEFINITION :2.6 (EXPECTED VALUE)

Let Y be a fuzzy variable. The **expected value** of Y is defined as

$$E[Y]=\int_0^{\infty} Cr\{Y \geq r\}dr - \int_{-\infty}^{0} Cr\{Y \leq r\}dr(2)$$

This integrals is finite.

Particularly, for non negative fuzzy variable Y, $E[Y] = \int_0^{\infty} Cr\{Y \geq r\}dr$.and let Y be a continuousnonnegative fuzzy variable with membership function μ is decreasing on $[0,\infty)$,then

$Cr\{Y \geq r\} = \frac{\mu_Y(r)}{2}$for any$r > 0$and $E[Y] = \frac{1}{2}\int_0^{\infty} \mu_Y(r)dr$

DEFINITION :2.7 (ARCHIMEDEAN)

A t-norm T is said to be **Archimedean** if : $T(x, x) < x$for all $x\in (0, 1)$.

DEFINITION :2.8 (T-INDEPENDENT)

Let T be a t-norm. A family of fuzzy variables $\{Y_i, i\in I\}$ is called **T-independent**. if for any subset $\{i_1,i_2,\ldots,i_n\}\subset I$ with $n \geq 2$,

$Pos\{Y_{i_k} \in B_k, k=1,2,\ldots,n\} = T_{k=1}^{n} Pos\{Y_{i_k} \in B_k\}$(4)

for any subsets $B_1, B_2, \ldots, B_n$ of $\Re$.Particularly, fuzzy variables $Y_1, \ldots, Y_n$ are T-independent if

$Pos\{Y_k\in B_k, k = 1, 2, \ldots, n\}=T_{k=1}^{n} Pos\{Y_k \in B_k\}$,(5)

For any subsets $B_1, B_2, \ldots, B_n$ of $\Re$. Furthermore, we say two families of fuzzy variables $\{Y_i, i\in I\}$ and$\{z_j, j\in J\}$ are mutually T-independent if for any $\{i_1,i_2,\ldots,i_n\} \subset I$ and $\{j_1,j_2,\ldots,j_n\}\subset J$ with $n,m\geq 1$,fuzzy vectors$\{Y_{i_1},Y_{i_2},\ldots,Y_{i_n}\}and\{Z_{j_1},Z_{j_2}\ldots,Z_{j_n}\}$are T-independent.

DEFINITION :2.9 (RENEWAL PROCESSES)

If the sequence of nonnegative random variables $\{X_1, X_{2,\ldots}\}$ is independent and identically distributed ,then the counting process $\{N(t), t \geq 0\}$ is said to be **renewal processes**.For any sequence of i.i.d random variable and them define a counting process as

$N(t) = \max\{n: S_n \leq t\}$with $\sum_{j=1}^{n} X_j = S_n$

The definition implies

(a) $N(t) \geq 0$

(b)$N(t)$ is integer valued

(c)if $s < t$, then $N(s) \leq N(t)$

(d)For $s < t, N(t) - N(s)$ equals the number of events in (s,t]

DEFINITION :2.10 (RENEWAL REWARD PROCESSES)

Consider a renewal process $\{N(t), t \geq 0\}$ having interarrival times $\{X_1, X_{2,\ldots}\}$ and suppose that each time a renewal occurs we receive a reward.We denote byR_n,the reward earned at the time of the nth renewal.We assume that the R_n,$n \geq 1$,are i.i.drandom variable.If we let

$$R(t) = \sum_{n=1}^{N(t)} R_n$$

Then $R(t)$repesents the total reward earned by time t.

APPLICATION OF COMPUTING THE AVERAGE REWARD IN A SERVICE SYSTEM

For fuzzy variables $Y_k, 1\leq k\leq m$ with possibility distributions $\mu_k, 1\leq k\leq m$ respectively and a function $g : \Re^m\rightarrow\Re$,the possibility distribution of $g(Y_1,Y_2,\ldots,Y_m)$ determined by the possibility distributions $\mu_1, \mu_2,\ldots, \mu_m$ via the following generalized extension principle ,

$$\mu_{g(Y_{1,\ldots,}Y_m)}(z) = \sup_{X=g(X_1,X_2,\ldots,X_m)} T^{Tn}_{k=1}\mu_k(X_k)$$

Where T can be any general t-norm .For a general t-norm T, by extension principle(6).The Possibility distribution of the arithmetic mean

$\left(\frac{Y_1+Y_2+\cdots+Y_n}{n}\right)$is $\mu_{\frac{1}{n}(Y_1+Y_2+\cdots+Y_n)}(z) = \mu_{(Y_1+Y_2+\cdots+Y_n)}(nz)$

$= \sup_{X_1+X_2+\cdots+X_n=nz} T(\mu_{Y_1}(X_1), \dots, \mu_{Y_n}(X_n))$(9)

furthermore, if Tis a continuous Archimedean t-norm with additive generator f from (3) and (9) ,the possibility distribution of$\left(\frac{Y_1+Y_2+\cdots+Y_n}{n}\right)$ can be determined by ,

$$\mu_{\frac{1}{n}(Y_1+Y_2+\cdots+Y_n)}(z) = \sup_{X_1+X_2+\cdots+X_n=nz} f^{[-1]}\left(\sum_{k=1}^{n} f(\mu_{Y_k}(X_k))\right)$$

$$= f^{[-1]}\left(\inf_{X_1+X_2+\cdots+X_n=nz} \sum_{k=1}^{n} f(\mu_{Y_k}(X_k))\right) \qquad (10)$$

Consider the concept of a multi-service station. Assume that the system offers six different types of services, and that users will seek out service I with probability p i, where p i=1/6, i=1,2,...,6. Customers are saved separately, and the service time T i (min) offered by service I and the corresponding cost C i ($) of service I in station are assumed to be positive triangular fuzzy variables as indicated in Table 1. The service time T i and the cost C i for each service I are considered to be T-independent. We'll calculate the multi-service station's long-term expected rewards using the t-norm T as a Yager t-norm T Y, >1.

Let ξ_k be the interarrival time between the $(k-1)$th and kth customers requesting services, and η_k be the reward gained from the kth customers, k = 1,2,… We know that the kth customer's requested service and the related reward are stochastic, but the service time and cost of each service in the station are fuzzy, and are characterised by triangular fuzzy variables T i and C i, respectively, for i=1,2,...,6. Taking into account all situations, the inter arrival times$\{\xi_k\}$ and rewards $\{\eta_k\}$ can be considered as two sequences of fuzzy random variables.

The distributions of the inter arrival times ξ_k and rewards η_k, k = 1,2,… can be presented as follows:

$$\xi_k \sim \begin{pmatrix} T_1T_2T_3 \; T_4T_5T_6 \\ p_1p_2p_3 \; p_4p_5p_6 \end{pmatrix} \text{ and } \eta_k \sim \begin{pmatrix} C_1C_2C_3 \; C_4C_5C_6 \\ p_1p_2p_3 \; p_4p_5p_6 \end{pmatrix} \text{ respectively.}$$

Table 1

service time and cost

Service i	**service time T_i (min)**	**cost C_i($)**
1	$T_1 = (2,3,5)$	$C_1 = (10,50,60)$
2	$T_2 = (3,4,6)$	$C_2 = (30,70,80)$
3	$T_3 = (4,5,7)$	$C_3 = (20,60,70)$
4	$T_4 = (5,6,8)$	$C_4 = (40,80,90)$
5	$T_5 = (6,7,9)$	$C_5 = (60,100,110)$
6	$T_6 = (7,8,10)$	$C_6 = (50,90,100)$

Table :2

Distribution of the random parameters U_k and V_k

Service i	**Realization $U_k(\omega_i)$**	**Realization $V_k(\omega_i)$**	**Probability**
1	$U_k(\omega_1) = 3$	$V_k(\omega_1) = 50$	1/6
2	$U_k(\omega_2) = 4$	$V_k(\omega_2) = 70$	1/6
3	$U_k(\omega_3) = 5$	$V_k(\omega_3) = 60$	1/6
4	$U_k(\omega_4) = 6$	$V_k(\omega_4) = 80$	1/6
5	$U_k(\omega_5) = 7$	$V_k(\omega_5) = 100$	1/6
6	$U_k(\omega_6) = 8$	$V_k(\omega_6) = 90$	1/6

The total service time S_n for the first n customers is calculated by $S_n = \xi_1 + \cdots + \xi_n$, and the total number N(t) of customers who have been served and the total reward C(t) by time t, are given by

$$N(t) = \max\{n > 0 | 0 < S_n \le t\}$$

And

$$C(t) = \eta_1 + \cdots + \eta_{N(t)},$$

Respectively. Given the above distributions of fuzzy random inter arrival times $\{\xi_k\}$ and $\{\eta_k\}$, without losing any generality, we assign values of two i.i.d random variable sequences $\{U_k\}$ and $\{V_k\}$ on probability space Ω as shown in table 2. Hence, the distributions of each pair of ξ_k and η_k for k = 1,2,… can be rewritten as

$\xi_k(\omega) = (U_k(\omega) - 1, U_k(\omega), U_k(\omega) + 2)$ (min)

And

$$\eta_k(\omega) = (V_k(\omega) - 40, V_k(\omega), V_k(\omega) + 10)(\$)$$

For any $\omega \in \Omega$.

Therefore, we can find the possibility function $\Pi_1 = (-1,0,2)$. Furthermore, since

$$\Pi_1(x) = \begin{cases} x + 1, & \text{if} x \in [-1,0] \\ 1 - \frac{x}{2}, & \text{if} x \in [0,2] \\ 0, & \text{otherwise} \end{cases}$$

We have,

$$f_\lambda^Y \circ \Pi_1(x) = \begin{cases} (-x)^\lambda, & \text{if} x \in [-1,0] \\ \left(\frac{x}{2}\right)^\lambda, & \text{if} x \in [0,2] \\ 1, & \text{otherwise,} \end{cases}$$

Which is the convex function on [−1, 2]. Thus, we have $Co(f_\lambda^Y \circ \Pi_1)(x) = f_\lambda^Y \circ \Pi_1(x) > 0$ for any $x \epsilon [-1,2]$. Similarly, we can get $Co(f_\lambda^Y \circ \Pi_R)(x) > 0$ for any nonzero $x \epsilon [-40,10]$.

As a consequence, by theorem (4.6), we can calculate that the average reward earned per minute by this service station in the long run is

$$\lim_{t \leftarrow \infty} \frac{E[C(t)]}{t} = \frac{\sum_{i=1}^{6} p_i V_1(\omega_i)}{\sum_{i=1}^{6} p_i U_1(\omega_i)} = 13.6\ (\$).$$

CONCLUSION

In this study, we investigated a fuzzy random renewal reward process under the T-independence associated with continuous Archimedean t-norms, and got the following new conclusions. For the expected reward per unit time of the renewal reward process, we developed a fuzzy random renewal reward theorem. In addition, we discovered the average reward in the service system.

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