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# Torsional and Torsional-Flexural Buckling Strength of Column

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A B S T R A C T

In the present study the torsional-flexural buckling is explained with the mathematical derivations. Energy principal are used in these derivations. From the analytical help the various modes of buckling are explained.It is found that the singly symmetric section will fail by flexural mode and if load eccentricity present the it can buckle by torsional-flexural mode and Non-symmetric sections always buckle buy torsional-flexural mode in general. However very long column can bend Euler' buckling mode but distortion sets in and lead to torsional-flexural buckling and this should be considered in design.

Keywords: Torsional-flexural buckling, Colmn bending , buckling load

## 1. Introduction

The flexural buckling of the column happens by bending about one of the symmetric axes of the section. In case of the plate or a strip when loaded in plane at the ends as shown in Fig. P1 (a) the plate bends about its weaker axis and hence buckling initiated. After buckling the plate cannot resist any load and hence collapse will occur.

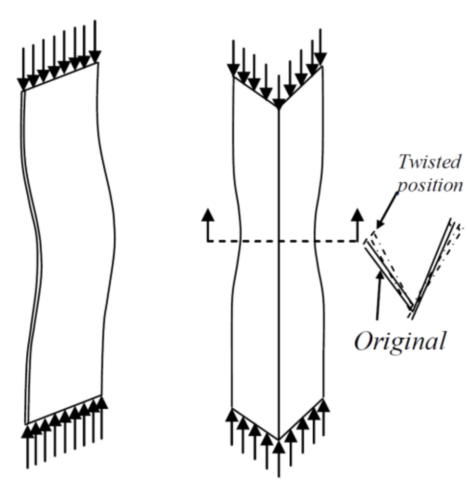


*Fig.P1 (a) Plate with unsupported edges*   *Fig.1 (b) Folded plate twists under axial load*

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In case if the same plate is bent along the longitudinal direction the direction of load axis then the plate will bend but retain its position and do not enter in the further deformation.

Close examination of the section will reveal that the outer edges would have bent more than the inner edges. This is because of the restraint received from one plate to another plate. Hence the angle shaped bent up plate is more efficient in carrying the axial load than the plain strip of plate. Further the deflection directions of the plates at the free edges for both plates are the same. This indicates that the cross section has swayed or rotated or twisted which is an indication torsion sets in the section. The centre portion of the section is restrained from other part of the section and hence the section to twist and buckle.

When section twist and buckle its load carrying capacity is very much reduced from flexural buckling strength. The fact that when a section is loaded at the shear centre torsion or twist cannot happen is not applicable to thin walled, mono-symmetric and point symmetric sections. This is because for thin angle sections the shear centre lies away from the sectional geometry. For cruciform section the C.G and Shear Centre lies in the same point the section rotate about the shear centre hence torsional buckling happens for intermediate columns. Any general sections the C.G and Shear Centre do not coincide and symmetry also do not exists hence torsion sets in and flexural buckling also happen which is termed as torsional-flexural buckling

## 2. Torsional-Flexural Buckling

To investigate the torsional flexural buckling load or the critical load for commonly used sections, we need to make some assumptions as follows:

1. The section is arbitrary in shape and open cross section.
2. The deflections are very small . The material is strained within the elastic limits and the buckling is elastic buckling.
4. The governing differential equations are integrated to obtain the solution with boundary conditions.
5. Ritz method is used to get the solution by minimizing the energy.
6. Load eccentricities with respect to both axes exist.

Consider a torsional flexural buckling of an arbitrary shape as in Fig. 2P. The torsional and flexural buckling always associated with sectional rotation or twisting and flexural buckling of bending. To obtain a closed form solution the buckling combination can be viewed as displacement on the transverse axes in both direction and a rotation. The shear centre is denoted by “O”. The coordinate axes X and Y are the transverse axis and the Z- axis is along the longitudinal axis of the member. It is also assumed that the X and Y axis are the principal axes of the member.

With reference to shear centre let the distance of centroidal axes are $x_0$ and $y_0$ along the X and Y directions. Due to buckling let the mid section of the member deflect a magnitude of u and v along the axes and rotate φ about the shear centre. Since the distortional buckling is not considered the section is assumed for no distortion.

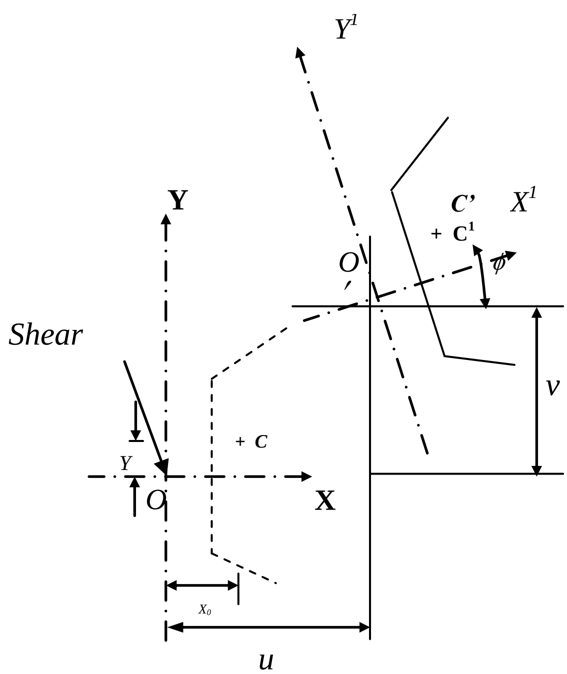


***Fig. P2 Torsional -flexural buckling deformations.***

Member end conditions:

The member is assumed to be hinged at both ends hence translation or displacement are zero at the ends.

$u = v = 0$ at $z = 0$ and $\ell$

$$\frac{d^2u}{dz^2} = \frac{d^2v}{dz^2} = 0 \ \ at \ \ z=0 \ \ and \ \ \ell$$

The hinge at the member ends can allow the rotational freedom and hence the torsional moments are zero at the ends

$$\phi = \frac{d^2\phi}{dz^2} = 0 \quad at \quad z=0 \ \ and \ \ \ell$$

To obtain a solution trial functions are considered satisfying the boundary conditions

$$u = C_1 \ \text{Sin} \frac{\pi z}{\ell}$$

$$v = C_2 \ \text{Sin} \frac{\pi z}{\ell}$$

$$\varphi = C_3 \ \text{Sin} \frac{\pi z}{\ell}$$

$$V = -\int_A \Delta_b \, \sigma \, dA$$

The solution technique is by minimizing the strain energy of the buckled system.
The energy components are due to:

i. Flexural bending energy along the X-axis.
ii. Flexural bending energy along the Y-axis.
iii. Plane shear stress energy due to St.Venant.
iv. Warping stress generated due to moment of the longitudinal fibres.

For all the four component the energy terms are summed up and given below:

$$U = \frac{1}{2}\int_0^\ell EI_y \left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{1}{2}\int_0^\ell EI_x \left(\frac{d^2v}{dz^2}\right)^2 dz$$

$$+ \frac{1}{2}\int_0^\ell GJ \left(\frac{d\varphi}{dz}\right)^2 dz + \frac{1}{2}\int_0^\ell E\Gamma \left(\frac{d^2\varphi}{dz^2}\right)^2 dz$$

"U" represent the total strain energy of the system.
$J$ = torsional constant
$\Gamma$ = warping constant

One solution for this equation can be obtained by substitution of the assumed trial function in the strain energy equation. This equation can be simplification as follows:

$$U = \frac{1}{4}\frac{\pi^2}{\ell}\left[\begin{array}{l} C_1^2 \ \dfrac{EI_y\pi^2}{\ell^2} + C_2^2 \ \dfrac{EI_x\pi^2}{\ell^2} \\ + C_3^2\left( GJ + \dfrac{E\Gamma\pi^2}{\ell^2}\right) \end{array}\right] \qquad (13)$$

The strain energy equation can be added to the potential energy of the system. The potential energy is equal to the load multiplied by the distance moved. Or this should be equal to the load multiplied by the axial shortening of the member.

Potential energy is given by

If $\Delta_b$ is equal to the difference between the arc lengths and the chord length $L$ of the fibre.
i.e. $\Delta_b = S - L$ (Fig. P3)

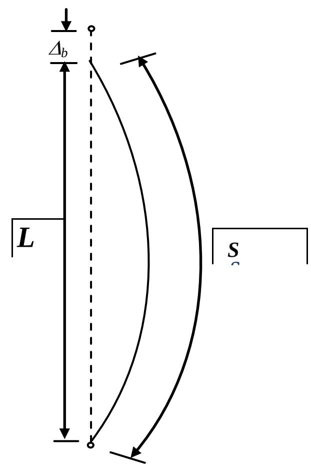


***Fig. P3 Axial shortening of longitudinal fibre due to bending***

Hence total external potential energy is equal to P x $\Delta_b$ where P is the applied load.
The external potential energy of the system is given by ( "$V$") is modified with normally used notations.

$$V = -\frac{P\pi^2}{4\ell}\begin{pmatrix} C_1^2 + C_2^2 + C_3^2 r_0^2 \\ -2C_1C_3y_0 + 2C_2C_3x_0 \end{pmatrix}$$

$C_1, C_2, C_3, x_0, y_0, r_0$ – all notations as explained in previous sections.

Substituting the Euler's buckling values and torsional buckling values:

$$P_y = \frac{\pi^2 EI_y}{\ell^2};\ P_x = \frac{\pi^2 EI_x}{\ell^2} \quad and\ P_\phi = \frac{1}{r_0^2}\left(GJ + \frac{E\Gamma\pi^2}{\ell^2}\right)$$

Total potential energy of the system is

$$U + V = \frac{\pi^2}{4\ell}\left\{\begin{matrix} C_1^2\left(\frac{\pi^2 EI_y}{\ell^2} - P\right) + C_2^2\left(\frac{\pi^2 EI_x}{\ell^2} - P\right) \\ + C_3^2 r_0^2\left(\frac{1}{r_0^2}\left(GJ + \frac{E\Gamma\pi^2}{\ell^2}\right) - P\right) \\ + 2C_1C_3Py_0 - 2C_2C_3Px_0 \end{matrix}\right\}$$

Using the notation $Py$, $Px$, and $P\varphi$

$$U + V = \frac{\pi^2}{4\ell}\begin{bmatrix} C_1^2\left(P_y - P\right) + C_2^2\left(P_x - P\right) \\ + C_3^2 r_0^2\left(P_\phi - P\right) + 2C_1C_3Py_0 \\ - 2C_2C_3Px_0 \end{bmatrix}$$

The total potential energy $(U+V)$ can be differentiated and equated to any stationary values like $C_1$, $C_2$ and $C_3$ to solve the conditions for the equation.

Then we get,

$$\frac{\partial(U+V)}{\partial C_1} = C_1\left(P_y - P\right) + C_3\left(Py_0\right) \qquad = 0$$

$$\frac{\partial(U+V)}{\partial C_2} = C_2\left(P_x - P\right) - C_3\left(Px_0\right) \qquad = 0$$

$$\frac{\partial(U+V)}{\partial C_3} = C_1\, Py_0 - C_2\, Px_0 + C_3\, r_0^2\left(P_\varphi - P\right) \qquad = 0$$

$$\begin{bmatrix} P_y - P & 0 & Py_0 \\ 0 & P_x - P & -Px_0 \\ Py_0 & -Px_0 & r_0^2(P_\phi - P) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

This is linear simultaneous equation with constant coefficients and hence results can be obtained by equating the coefficient determinant to zero. i.e

$$\begin{vmatrix} P_y - P & 0 & Py_0 \\ 0 & P_x - P & -Px_0 \\ Py_0 & -Px_0 & r_0^2\,(P_\varphi - P) \end{vmatrix} = 0$$

The equation reduce to,

$$(P_y - P)(P_x - P)(P_\phi - P) - \left(P_y - P\right)\frac{P^2 x_0^2}{r_0^2} - (P_x - P)\frac{P^2 y_0^2}{r_0^2} = 0$$

The reduced equation is a third order or cubic equation and hence should have three roots corresponding to critical load.

These three loads will dictate the values of three buckling modes.

Doubly symmetric sections the centriodal axis coincides with the shear centre. Hence

$x_0 = 0$ and $y_0 = 0$

$\therefore\ (P_y - P)\ (P_x - P)\ (P_\varphi - P) = 0$

This equation has three roots, namely,

$$\left.\begin{aligned} P &= P_x = \frac{\pi^2 EI_x}{\ell^2} \\ P &= P_y = \frac{\pi^2 EI_y}{\ell^2} \end{aligned}\right\}$$

– These represent Euler loads by buckling about the $x$ and $y$ axes

$$P = P_\phi = \frac{1}{r_0^2}\left(GJ + \frac{E\Gamma\pi^2}{\ell^2}\right)$$

– This represents the Torsional buckling load.

The geometrical shape of the cross section mostly govern the buckling mode for intermediate range of columns. For doubly symmetric shape the shear centre distance is zero with respect to cetriodal axes only the Euler's buckling value govern.
Channel, equal angle sections which are singly symmetric the component $y_0 = 0$

Then,

$$y_0 = 0 \qquad P = P_y = \frac{\pi^2 E I_y}{\ell^2}$$

(This represents Euler Buckling Load)

$$\therefore (P_x - P)(P_\phi - P) - \frac{P^2 x_0^2}{r_0^2} = 0$$

This equation is a quadratic equation and hence can have two roots. The least value of this quadratic equation can be worked out as:

$$P_{TF} = \frac{1}{2k}\left[P_\varphi + P_x - \sqrt{(P_\varphi + P_x)^2 - 4kP_\varphi P_x}\right]$$

$$k = \left[1 - \left(x_0 / r_0\right)^2\right]$$

Where the notation $P_{TF}$ represent the torsional-flexural buckling value.

## 3. Modes of Buckling

Concentrically loaded members always buckle. The buckling for long column is by flexure or called as Euler's buckling. There are three modes of buckling:
1. Flexural buckling on strong or weak axis.
2. Torsional buckling about the longitudinal axis.
3. Tosional-flexural buckling.

Doubly symmetric sections buckle flexural mode except the cruciform section in the intermediate range. Cruciform section will fail by torsional mode if the column is in the intermediate length, but fail by flexural mode if the column is in the Euler's column range. Singly symmetric section will fail by flexural mode and if load eccentricity present the it can buckle by torsional-flexural mode. Non-symmetric sections always buckle buy torsional-flexural mode in general. However very long column can bend Euler' buckling mode but distortion sets in and lead to torsional-flexural buckling and this should be considered in design.

## 4. Conclusion

The torsional-flexural buckling is explained with the mathematical derivations. Energy principal are used in these derivations. From the analytical help the various modes of buckling are explained.

## References

[1] Timoshenko S.P. and Gere, J.M: Theory of Elastic Stability, McGraw Hill Kogakusha Ltd., New York.
[2] Chajes, A.: Principles of structural Stability Theory, Prentice Hall, New Jersey,1974
[3] Bureau of Indian Standards, New Delhi, 2007.IS:800-2007. 'Code of Practice for General Construction in Steel'